

On some information-geometric aspects of Hawking radiation

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Abstract

This paper illustrates the resemblance between the information-geometric structures of probability spaces and that of the discrete spectrum for Hawking radiation. The information geometry gives rise to a reconstruction of the standard formalism of quantum mechanics, while the discrete spectrum of Hawking radiation contributes to the semiclassical unitary evolution of Hawking radiation. If more realistic models of Hawking radiation are chosen, the information-geometric structures of the probability space for Hawking radiation can be constructed from some physical considerations. The constructed quantum formalism is consistent with both the unitary evolution of Hawking radiation in the semiclassical picture and the topology change of fuzzy horizons. These aspects of Hawking radiation can be connected to some general convictions of quantum gravity such as holography. A comparison with the fuzzball proposal shows the limitation and effectiveness of this construction. We conclude that these information-geometric aspects show some possible ways bridging the gap between semiclassical models and quantum gravity.

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I. INTRODUCTION

Quantum field theory in curved spacetime predicts many intriguing physical results, such as the Hawking radiation and the Unruh effect[1]. However, the ideal mathematical abstractions in the field-theoretic analysis prevent it from corresponding directly to the phenomena in realistic physical world. For instance, boxes with ideally perfect mirrors as box walls experiencing the Unruh effect will cause the alleged self-accelerating box paradox, which will be greatly suppressed if a more realistic model of mirror is chosen[2]. In the case of Hawking radiation, the information loss paradox is a similar problem. Ideally field-theoretic evaluations of the Bogoliubov coefficients show that the radiation spectrum is purely thermal and hence the nonthermal information is lost if the black hole evaporates completely[3]. But if we consider the Hawking radiation as a tunneling process and take into account the back reaction of the radiation[4], we can not only easily obtain the related quantities in black hole physics, but also have a more realistic picture of Hawking radiation with spectrum slightly differing from the original field-theoretic calculation. In particular, recent works have argued that this slight difference allows the *hidden messengers*[5], the correlations between successive radiations, to recover nonthermal information from black holes. Further, it is shown that the average effects of the dynamical processes of Hawking radiation lead to an effective formulation of the black hole quasi-normal modes that takes a discrete and countable form such that a radiating black hole looks like an excited atom[6], which results in a unitary evolution model for Hawking radiation described by a time-dependent Schrödinger equation[7].

All these advances listed above are carried out in a picture that has been modified constantly to be more realistic instead of a picture employing certain delicate, of course more undetectable, quantum gravity model. One might argue that these reality-oriented modifications might make the picture become more classical, whence the assertion that the information is conserved during the Hawking radiation is unconvincing at the semi-classical level. However, in spite of this drawback, the phenomenologically unitary evolution of Hawking radiation at this level should reveal some features of the underlying full quantum theory. One foremost feature in [6] is the discreteness of the horizon area as the function of the overtone number of quasi-normal modes, which is consistent with various quantum gravity models where the spacetime is fundamentally discrete[8].

On the other hand, Hawking radiation is theoretically predicted and can be detected *in principle*. While *in practice*, this prediction is still far from being confirmed. Till now the most famous detecting proposal is the Unruh-DeWitt detector[9]. The Unruh-Dewitt detector is a theoretically designed point detector interacting locally with a quantum field originally to prove the existence of Unruh effect for detectors moving in Minkowski spacetime, and a static detector can be utilized in black hole spacetime to detect the thermal spectrum of Hawking radiation[10]. By adding some realistic considerations to the detector, such as the spatial profile of the detector and the finite time effect, we can get results consistent with locality by some peculiar regularizations[11]. More realistic models of the spatially smeared detectors can be constructed by putting detecting atoms in standard QED interactions[12]. In fact, Hawking radiation can be attributed to the Lamb shifts of the collapsing atoms provided that only the high-energy modes can be really detected[13]. With these realistic models in hand, we can phenomenologically identify the tunneling rate of Hawking radiation with the spectral response (or the transition rate) of the Unruh-DeWitt detector by this atom-atom model correspondence.

For all these reasons we observe that, in the identification said above, the probability space of the tunneling rates (or the responses) has alike geometric structures of those analyzed in [14]. In [14], a probability space $\{P^i; i = 1, \dots, n, \sum_i P^i = 1\}$ with an information metric

$$ds^2 = G_{ij}dP^i dP^j = \frac{\alpha}{2P^i} \delta_{ij} dP^i dP^j, \quad (1)$$

can be set in motion by introducing S^i , the canonical conjugations of P^i , to saturate a symplectic structure. Subsequent attempts to extend the n-dimensional information metric (1) to the 2n-dimensional phase space $\{P^i, S^i\}$ require a Kähler structure on the phase space. After changing $\{P^i, S^i\}$ to the complex coordinates $\{\psi^i, \bar{\psi}^i\}$ via the Madelung transformation, the coordinate variables take the forms of wave functions in standard quantum mechanics. And the underlying Hilbert space can be constructed from the symplectic form on this Kähler manifold. The interpretation of the auxiliary quantities S^i is controversial though.

In this paper, we first, by considering special cases in which the dimension n is odd, find that part of S^i might carry the information of the topological structures of the pertinent Kähler manifold. Then in the context of evaporating black holes, we find that the standard quantum mechanical structures constructed from the probability space of the

tunneling rates of Hawking quanta (or the responses of detectors) are consistent with the aforementioned quantum unitarity of Hawking radiation in the semiclassical models, since with S^i appearing on the exponent we can only detect the probability space $\{P^i\}$. Hence the effectively unitary quantum evolution can be envisioned as an emergent result from the coarse graining of a more fundamental fine-grained theory. We compare this emergent result with 't Hooft's proposal of quantum determinism[15] . And the S-matrix in this reconstructed Hilbert space is similar to that proposed in [16] for Hawking radiation. Furthermore, these results are compared with the predictions of the fuzzball proposal, which exhibits limitations of this construction. Altogether, we conclude that these information-geometric aspects of Hawking radiation implies some possible connections between the semiclassical results and the deeper quantum gravity predictions.

This paper is organized as follows: In section II, after briefly revisiting the information-geometric reconstruction of quantum mechanics, we show that part of S^i are related to the nonvanishing first Chern class of the resultant Kähler manifold. In section III, the reconstruction procedure is applied to evaporating black holes in the semiclassical picture and the physical comprehensions of this procedure are pointed out. In section IV, the reconstruction is compared with some general implications from quantum gravity, such as holographic principle, the S-matrix of Hawking radiation and fuzzballs. Section V concludes.

II. INFORMATION-GEOMETRIC RECONSTRUCTION OF QUANTUM MECHANICAL FORMALISM

In this section, we first briefly review the geometrically inspired reconstruction of quantum mechanics from the metric (1)[14]. There are reconstructions based on physical postulates which also start from (1)[17, 18], but the approach in [14] can be most easily applied to the case of Hawking radiation. Then we give some remarks concerning the interpretation of the reconstructed formalism.

A. Reconstruction

Let us begin with a probability space $\{P^i; i = 1, \dots, n, \sum_i P^i = 1\}$ with the information metric (1). Now consider the case in which the probabilities change with time. Suppose that the evolutions of the probabilities can be generated by an action principle and hence introduce the $\{S^i\}$ conjugate to $\{P^i\}$. The Poisson brackets are defined in the usual sense with the symplectic matrix

$$\Omega = \begin{pmatrix} 0 & \mathbf{1}_{n \times n} \\ -\mathbf{1}_{n \times n} & 0 \end{pmatrix}. \quad (2)$$

Then the phase space $\{P^i, S^i\}$ becomes a symplectic manifold with the canonical symplectic structure $\omega = \sum_{i=1}^n dS^i \wedge dP^i$.

Since the metric G_{ij} in (1) is defined on the n -dimensional configuration space $\{P^i\}$ only, whereas the evolving phase space $\{P^i, S^i\}$ is $2n$ -dimensional, we have to extend G_{ij} to the $2n$ -dimensional g_{ab} . Consistency of the symplectic structure and the metric structure requires the following (see [14] for detail)

$$\Omega_{ab} = g_{ac} J^c_b, \quad (3)$$

$$J^a_c g_{ab} J^b_d = g_{cd}, \quad (4)$$

$$J^a_b J^b_c = -\delta^a_c. \quad (5)$$

That is, the manifold must have a Kähler structure with complex structure J_{ab} . As a result, the extended metric g_{ab} and the complex structure J_{ab} become

$$[g_{ab}] = \begin{pmatrix} G & A^T \\ A & (1 + A^2)G^{-1} \end{pmatrix}, \quad (6)$$

$$[J^a_b] = \begin{pmatrix} A & (1 + A^2)G^{-1} \\ -G & -GAG^{-1} \end{pmatrix}, \quad (7)$$

where $G = [G_{ij}]$ and $A^T = GAG^{-1}$. It can be further shown that the information metric (1) is the only metric invariant under the congruent embedding by a Markov mapping, which constrains that A must be a constant matrix $A = a\mathbf{1}_{n \times n}$ [14].

Now we can perform a (modified) Madelung transformation to the complex coordi-

nates,

$$\begin{aligned}\psi^k &= \sqrt{P^k} \exp[i(\frac{\Lambda}{\alpha} S^k - \gamma \ln \sqrt{P^k})], \\ \bar{\psi}^k &= \sqrt{P^k} \exp[-i(\frac{\Lambda}{\alpha} S^k - \gamma \ln \sqrt{P^k})],\end{aligned}\tag{8}$$

where $\Lambda = \frac{1}{1+A^2}$, $\gamma = \frac{-A}{1+A^2}$ and $k = 1 \dots n$. Consequently,

$$[\Omega_{ab}] = \begin{pmatrix} 0 & i\alpha\Lambda^{-1}\mathbf{1} \\ -i\alpha\Lambda^{-1}\mathbf{1} & 0 \end{pmatrix},\tag{9}$$

$$[g_{ab}] = \begin{pmatrix} 0 & \alpha\Lambda^{-1}\mathbf{1} \\ \alpha\Lambda^{-1}\mathbf{1} & 0 \end{pmatrix},\tag{10}$$

$$[J^a_b] = \begin{pmatrix} -i\mathbf{1} & 0 \\ 0 & i\mathbf{1} \end{pmatrix}.\tag{11}$$

Let $\frac{\alpha}{\Lambda} = \hbar$, then (8) look similar to the wave functions in standard quantum mechanics. If $A = 0$, then it is reduced to the flat-space case in which with the second term on the exponent vanishes (8) take the same form of the wave functions in standard quantum mechanics. The inner product defining a Hilbert space can then be constructed also in this flat-space case

$$\langle \psi | \varphi \rangle = \frac{1}{2} \sum_i (\psi^i, \bar{\psi}^i) (g + i\Omega) \begin{pmatrix} \varphi^i \\ \bar{\varphi}^i \end{pmatrix} = \sum_i \bar{\psi}^i \varphi^i.\tag{12}$$

The canonical equations of motion in this Hilbert space can be obtained immediately.

B. Remarks

We first remark that the complex vector space \mathbb{C}^n has a natural Kähler structure[19]. What is nontrivial in the above reconstruction is that we start from the real probability space but arrive at a Kähler manifold. Once we have obtained the Kähler structure, the transformation to the complex coordinates is trivial.

Secondly, we discuss the topological meaning of some part of S^i . On the one hand, the allowed transformation group preserving the probability normalization and the information metric is $O(2n)$ and hence the vectors in the probability space can be connected by curves lie on the unit sphere S^n [14]. On the other hand, the observables, say the expectation values, should be invariant under the $U(1)$ phase shift. If the dimension $n = 2k + 1$

is odd, then we can see that the probability space is just the complex projective space $\mathbb{C}P^k \simeq S^{2k+1}/U(1)$. It is well-known that $\mathbb{C}P^n$ is a Kähler manifold with the standard Fubini-Study metric[19]

$$ds^2 = \sum_{k=0}^n dz^k d\bar{z}^k - \left(\sum_{i=0}^n z^i d\bar{z}^i\right) \left(\sum_{j=0}^n \bar{z}^j dz^j\right), \quad (13)$$

where z^k, \bar{z}^k satisfying $\sum_k z^k \bar{z}^k = 1$ are coordinates on S^{2n+1} . The second term in (13) is similar to the correction term added to the information metric (1) in [17] identifying the disturbance of different preparations of the systems to the measurement results. The corrected metric also leads to the complex coordinates in [17],

$$\psi^i = \sqrt{P^i} \exp[i\phi^i]$$

where ϕ^i is just the degrees of disturbance identified by the correction term.

To elaborate on this point, we recall that $\mathbb{C}P^n$ is also a Kähler-Einstein manifold with the Ricci form ρ proportional to the Kähler form ω [19],

$$\rho = (2n + 2)\omega.$$

On a Kähler manifold the Ricci form ρ is identical to the first Chern class c_1 . In the case of $\mathbb{C}P^n$, the Kähler form and hence the first Chern class does not vanish. Therefore the holonomy group cannot be reduced to $SU(n)$ due to the topological obstructions. In analog to the interpretation of the phase factor in [17], the phase factor S^i here, at least part of it, carries the information of the topological structure of the pertinent Kähler manifold provided that the dimension of the probability space is odd. Since in the statistical interpretation of quantum mechanics the observed probability distribution is considered as a specific way of preparing the system instead of an intrinsic property of the system, the topology of the probability space can be traced back to the preparation of the system. Both the measurement context, a choice of basis vectors in Hilbert space, and the information about the topological structures of the underlying Kähler manifold, a specific preparation of the system, can not be seen from the quantum mechanical measurements that cancel the phase factors.

This corroborates the known fact that quantum states are rays in Hilbert space, while the relative phases can contribute to significant observable effects, i.e. the Berry phase. Studying quantum mechanics within $\mathbb{C}P^n$ has been established since long and the Berry

phases can be obtained by evaluating the parallel transport along the geodesics on it[20]. The $\mathbb{C}P^n$ of current concern is identical to that reduced from a Hilbert space of dimension $n + 1$ as is shown in [20]. However, noting that when the dimension of the probability space is even it cannot be reduced to a $\mathbb{C}P^n$, one sees that the above interpretation is not generically correct. Nevertheless, the even cases can be assumed to have trivial topology. We speculate that the dimension of the probability space might be constrained or rather identified by its topology. Disregarding this speculation, one can still give the following arguments. The invalidity of the use of the second Chern class for the $U(1)$ reduction of an even dimensional space has already been pointed out in [21]. The monopoles, as will be shown to be the case of current interest in the next section, are only possible when the space dimension is odd, whereas when the dimension is even, one has instanton solutions, which deserve further investigations especially in the tunneling picture of Hawking radiation.

Note that in [14] the phase factors S^i are related to the classical groups of motion as the momentum densities. This interpretation is of course desirable for the classical limit, however, it is not the whole story in quantum phenomena. For instance, after a measurement of a quantum state, what one obtains is the probability distribution P^i instead of S^i , but does one lose only the information of particle momenta? The answer is obviously no. In fact, the wave functions in the form of (8) are the projections of state vectors in a Hilbert space to specific representations in the orthodox interpretation. In many other representations the wave functions do not take the plane-wave form that supports momentum-density interpretation of S^i . But given the topological interpretation of S^i , how should we understand the peculiar form (8) that is absent in orthodox quantum formalism? Apart from the analogues with topological phases pointed out above, we can see that this interpretation is also supported by the universal coefficient theorem and the resultant principle of topological covariance[22]. Here the change in topology is described by the change in the coefficient groups in the cohomology $H^2(G, U(1))$ of the probability space, which are probed by particular experimental setups. The phase factors $\exp(iS^i)$ in (8) characterize the group extension $G/U(1)$ of the related group G by $U(1)$. By the universal coefficient theorem

$$0 \rightarrow \text{Ext}(H_1(G, R), U(1)) \rightarrow H^2(G, U(1)) \rightarrow \text{Hom}(H_1(G, R), U(1)) \rightarrow 0, \quad (14)$$

there exist different elements in $H^2(G, U(1))$ mapped into the same element in Hom , and the extension Ext characterizes different choices of coefficient structure R . Hence the measured probability distributions are identical for different states up to extension factors $\exp(iS^i)$. Since the group extension factors $\exp(iS^i)$ depends on the choice of the coefficient groups, we see that S^i indeed carry the information about the topological structures of the probability space.

III. ASPECTS OF HAWKING RADIATION

Now we can apply the above results to the situation of evaporating black holes emitting Hawking radiation. As mentioned above, the Hawking radiation spectrum is discrete in the effective tunneling model[6], which is consistent with the discrete spacetime in existing quantum gravity models[8], especially the discrete Hawking radiation spectrum due to the discrete horizon area[23]. The key ingredient here is the discrete or quantized horizon area $A = \sum_i N_i A_i$ for some possible area eigenvalues A_i , with which the emission rate of Hawking radiation in the semiclassical tunneling picture

$$\Gamma_{\text{out}} \sim \exp[\Delta S] = \exp[\Delta A/4] \quad (15)$$

becomes discrete. Notice that (15) is a probability rate, the probability of emission per unit time, instead of the probability. However, we further notice that in the unspecified actual probability space the different discrete probability rates indicate that this probability space is discrete and is changing or evolving with time. This is exactly the type of the probability space that we were discussing in section II with the trivial normalization. The information metric (1) can be considered as a consequence of the statistical distance between different probabilistic outcomes[24] and hence exists for this probability space.

Now that the probability space $\{P_{\text{out}}^i\}$ of Hawking radiation is evolving with time, we have to ask whether the laws of motion of this probability space is just what we have constructed in section II. At the first sight, the evolution of the Hawking radiation probability should be attributed to certain underlying physics and simply enforcing additional quantities such as S^i will not be useful at all. The difficulty here is that we still do not know the underlying quantum gravity very well. However, we can turn to the observation side to see what the Unruh-DeWitt detectors tell us.

Before we proceed it is worth noticing that the transition rate w of an Unruh-DeWitt detector is defined in terms of the time derivative of the response function $F(\omega, \tau)$ as[1]

$$w = \lambda^2 \mu^2 \dot{F}(\omega, \tau), \text{ with} \quad (16)$$

$$F(\omega, \tau) = \int_{\tau_0}^{\tau} d\tau' \int_{\tau_0}^{\tau} d\tau'' e^{-i\omega(\tau' - \tau'')} \cdot \langle 0 | \phi(\tau') \phi(\tau'') | 0 \rangle \quad (17)$$

if the monopole interaction $H_I = \lambda \mu \phi$ is assumed. After some calculations we can see that the transition rate takes the form of thermal spectrum

$$\dot{F}(\omega, 0) = \frac{1}{2\pi} \frac{\omega}{e^{2\pi\omega} - 1}, \quad (18)$$

which seems to be different from the tunneling rate (15). Indeed, the transition rate (18) has averaged over an ensemble of atoms in the detector, while the tunneling rate (15) is for a single Hawking particle. This can be reconciled by evaluating the spectrum corresponding to (15), as is carried out in [25]. If the condition of detailed balance is reached, (15) and (18) give the same Boltzmann factor

$$\frac{\dot{F}(\omega, \infty)}{\dot{F}(-\omega, \infty)} = \frac{\Gamma_{out}}{\Gamma_{in}} = \exp[-\beta\omega] \quad (19)$$

with temperature $1/\beta$.

In the light of this observation we expect the evolution of P_{out}^i can be inferred from that of \dot{F} . The evolutions of the detector's atoms are of course quantum mechanical and can be described by a recently presented approximate master equation for the density matrix of the detector[26]

$$\dot{\rho}_{mm} = \sum_{k \neq m} [\rho_{kk} w_{mk} - w_{km} \rho_{mm}] \quad (20)$$

in terms of the matrix elements with w given by (16). As we can see, the transition rate w here behaves like a Hamiltonian for the evolution of the detector (atoms). Since $w(\dot{F}, \cdot)$ is a function of \dot{F} 's or rather of F 's and other possible parameters, we could choose these unspecified parameters as quantities G 's conjugate to the F 's. Then by the fact[19] that once we have such a Hamiltonian function w a Hamiltonian vector field can be constructed as

$$X_w = \sum_{\omega} \left(\frac{\partial w}{\partial F_{\omega}} \frac{\partial}{\partial G_{\omega}} - \frac{\partial w}{\partial G_{\omega}} \frac{\partial}{\partial F_{\omega}} \right) \quad (21)$$

from which the Poisson brackets come out immediately. We thus have, as a direct consequence of the quantum mechanical equation of evolution (20), the desired symplectic

structure for the evolving probability space $\{F_\omega\}$ for the clicks of the detector. Correspondingly, we have the same symplectic structure for the probability space $\{P_{\text{out}}^i\}$ of Hawking radiation.

Starting from this symplectic structure of $\{P_{\text{out}}^i\}$, we can follow the discussion in section II step by step and obtain the complex coordinates of the pertinent Kähler manifold that are in the form of wave functions

$$\psi_{\text{out}}^i = \sqrt{P_{\text{out}}^i} \exp\left[\frac{i}{\hbar} S^i\right], \quad \bar{\psi}_{\text{out}}^i = \sqrt{P_{\text{out}}^i} \exp\left[\frac{-i}{\hbar} S^i\right], \quad (22)$$

with $i = 1, \dots, n$ the same as the probability space. The phase factors S^i , as is illustrated above, carry the information about the topological structures of the probability space. This still can be correct in the current case of black holes. Recall that Hawking radiation as topology changes of a fuzzy sphere S_F^2 was proposed in [27]. In this approach the fuzzy sphere Hilbert space is $(2J + 1)$ -dimensional and the selection rule for the area transition restricts the following $J \rightarrow J - 1/2$ (where J^a is the n -dimensional irreducible representation of $su(2)$ Lie algebra and J 's are half integers). We therefore see that the wave functions (22) can correspond to the wave functions M defined on the fuzzy sphere, and the alterations of dimensions between odd and even identify the changes of topology characterized by the phase factors. Then the process of Hawking radiation as topology change or the splitting of M into the main and the baby world with the degrees of freedom in the baby world inaccessible to observers in the main world, in the language of the above reconstruction, is a measurement of the quantum states to obtain the probabilities with the phase factors that characterize the topological structures being lost. As a consequence, observers in the main world can not detect the change in the Hilbert space dimension or the fine-grained topology. What can be detected is the coarse-grained probability space under the quantum mechanical evolution.

In order to show more details we first recall that the algebra Mat_n of $n \times n$ matrices is the structure algebra of S_F^2 , which can be generated, as is chosen above, by the $SU(2)$ irreducible representation J^a . In terms of eigenvectors of J^3

$$J^3 |m\rangle = m |m\rangle, \quad m = -j \dots j, \quad j = \frac{n-1}{2}, \quad (23)$$

we can define coherent states[28]

$$|\alpha\rangle = \left(\frac{1}{2} \sin \theta\right)^j \sum_{m=-j}^j \sqrt{\frac{(2j)!}{(j+m)!(j-m)!}} \left(\frac{\alpha}{R}\right)^{-m} |m\rangle \quad (24)$$

where $\alpha = R \tan(\theta/2) \exp[i\phi]$ and (R, θ, ϕ) are spherical coordinates. Then the fuzzy sphere S_F^2 contains n cells each of which supports a coherent state $|\alpha\rangle$. With these n coherent states one can choose orthonormal basis for polynomial functions on S_F^2 in terms of coupled oscillators $\{a_0^{(\dagger)}, a_1^{(\dagger)}\}$,

$$\sqrt{\binom{n}{k}} a_0^{n-k} a_1^k. \quad (25)$$

Then with the help of the projectors $p = |\psi_n\rangle \langle \psi_n|$ determining projective modules, with

$$|\psi_n\rangle = N \begin{pmatrix} a_0^n \\ \dots \\ \sqrt{\binom{n}{k}} a_0^{n-k} a_1^k \\ \dots \\ a_1^n \end{pmatrix}$$

being the basis vector that consists of (25), the first Chern class can be calculated as in [29],

$$c_1 = \frac{-1}{2\pi i} \int \text{Tr} p(dp)(dp) \neq 0$$

$$\xrightarrow{n \rightarrow \infty} k \in \mathbb{Z}. \quad (26)$$

Hence we see that the n cells on S_F^2 do carry topological charges that are in analog to the N-bit strings in each cell in [30]. And that the measurements change the topological charges is analogous to the measurements taking out a cell to get N classical bits. The phase factors are cancelled in the projector p and are undetectable in conventional analysis. However, the fuzzy line bundles or equivalently the projective modules are classified by the characteristic classes or topological charges, which corroborates the topological interpretations in the last section.

IV. IMPLICATIONS FROM THE RECONSTRUCTION

In this section, we discuss the implications for quantum gravity in the above reconstructed formalism. We compare the above results with simple models based on 't Hooft's

quantum determinism and black hole S-matrix. Finally a comparison with some results of the fuzzball proposal is presented.

Firstly, in the proposal that Hawking radiation can be attributed to the Lamb shifts of the collapsing matter[13], a collapsing black hole is modelled as infalling quantum oscillators interacting with external fields, which resembles the Unruh-DeWitt detectors. This explains why the atom-atom model correspondence will work in our analysis. Indeed, the quantum harmonic oscillator could be envisioned as an emergent result of underlying deterministic systems. In [31], by quantizing the following deterministic Hamiltonian using Faddeev-Jackiw technique

$$H = xp_y - yp_x, \quad (27)$$

one can get the reduced Lagrangian in the correct form of the linear harmonic oscillators,

$$L_R(\zeta, \dot{\zeta}) = \frac{1}{2}\dot{\zeta}^s \omega_{st} \zeta^t - \frac{1}{2a_1} p_\zeta^2 - \frac{a_1}{2} \zeta^2 \quad (28)$$

if the constraint $\phi = H - a_1(x^2 + y^2) = 0$ is chosen. Now in the above reconstruction, what an asymptotic observer can really detect is the probability distribution of Hawking radiation admitting the unitary quantum dynamics and hence the information about the underlying dynamics is lost. Therefore, our analysis of the information-geometric aspects of Hawking radiation supports the proposal that standard quantum mechanical structures can be considered as an emergent result of a deeper deterministic theory[15]. And the atom-atom model correspondence is valid only at the realistically semiclassical level with the reconstructed quantum states (with observables) representing the equivalence class of underlying states (with beables), which is exactly the original holographic principle.

Secondly, in the above reconstructed Hilbert space, we can try to construct the S-matrix for Hawking radiation. For a single tunneling of Hawking radiation from a Schwarzschild black hole, the gravitational back reaction contributes to a phase shift in the tunneling rate[4]

$$\Gamma_{\text{out}} \rightarrow \exp[4\pi\omega^2] \Gamma_{\text{out}}. \quad (29)$$

Then for states $|\psi_{\text{out}}^i\rangle$ in the reconstructed Hilbert space, the phase shift becomes

$$\exp[2\pi\omega^2]. \quad (30)$$

Consider an ingoing state $|\psi_{\text{in}}^i(\omega)\rangle$ that contributes to the Hawking radiation of energy ω . The outgoing state under back reaction is

$$|\psi_{\text{out}}^i(\omega)\rangle \rightarrow \exp[2\pi\omega^2] |\psi_{\text{out}}^i(\omega)\rangle. \quad (31)$$

The S-matrix is then

$$\langle\psi_{\text{out}}^i(\omega)|\psi_{\text{in}}^i(\omega)\rangle = N \exp[2\pi\omega^2] \quad (32)$$

for some normalization factor N . This is consistent with the S-matrix of black hole radiation presented in [16],

$$\langle\psi_{\text{out}}^i(\omega)|\psi_{\text{in}}^i(\omega)\rangle \sim \exp[i \int d\omega p_{\text{in}}(\omega) p_{\text{out}}(\omega)]. \quad (33)$$

Thirdly, an intriguing comparison with the fuzzball proposal for black holes can be made. The horizon-size horizonless fuzzballs are microstates of certain black holes in string theory. If all known black holes admit this microscopic structure, one has to find ways to connect it to the classical notions such as horizons. In [32] it is conjectured that fuzzballs can be formed from the collapsing shells via phase evolution

$$\sum_k c_k |E_k\rangle \rightarrow \sum_k c_k e^{-iE_k t} |E_k\rangle \quad (34)$$

which takes the initial special superposition to the general superposition that includes much more fuzzball states. This dephasing effect can be well illustrated by the fuzzball complementarity[33] where high energy infalling quanta with $E \gg T_{\text{BH}}$ collectively excite the fuzzballs and experience coarse-grained free fall, while quanta with $E \sim T_{\text{BH}}$ experience the fine-grained structures of fuzzballs and do not feel free fall. Hence, complementarity does not hold for Hawking quanta with $E \sim T_{\text{BH}}$ that encodes the black hole information and for a distant Unruh-DeWitt detector the unitary detection is not necessarily equivalent to knowing everything about the microscopic states. As a result, for high energy modes the complementarity holds and the Lamb shifts arguments also can be applied, which entails the reconstruction above. For low energy modes, although the cancelling of phases is possible it has nothing to do with the unitary evolution detected by a distant detector due to the invalidity of the complementarity. Indeed, in this case information is already carried out by Hawking quanta with $E \sim T_{\text{BH}}$ from fuzzballs but the observed unitary evolution does not reflect microscopic structures of fuzzballs, just as acted by an inverse map of (34), and hence the effective atom-like structure endures.

V. CONCLUSION

In this paper, we have discussed the information-geometric aspects of Hawking radiation. The procedure of reconstructing quantum mechanical formalism from information geometry is applied to the cases of evaporating black holes. We find that this reconstruction not only explains the successes of the semiclassical tunneling models but resembles other novel models of Hawking radiation. In addition, every step of this procedure can be based on adequate physical grounds. Connections to deeper quantum deterministic theory are pointed out, whereas comparison with the fuzzball proposal shows the limitations of this reconstruction. Part of the quantities conjugate to the probabilities in the above construction are shown to characterize the topological structures of the pertinent space.

In this reconstruction, Hawking radiation becomes a process similar to the emission of photons from a piece of burning coal[32]. The correlation arguments for information recovery in [5] are similar to the correlation of spins of emitted photons from coal, which explains its validity. Therefore the reconstruction in this paper provides possible ways to uncover deeper structures in quantum gravity.

Moreover, comparison with fuzzball model shows the limitations of this reconstruction. It has been argued that small corrections to the thermality of Hawking effect do entail unitary evolutions but fail to give correct statistical behavior of the entropy[34]. However, the analysis in this paper still holds at the semiclassical level. For more detailed microscopic description, the orthodox quantum mechanics might not be sufficient and an understanding of deeper structures is required.

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